ALGEBRAIK SHAKLDAGI KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR.

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KIRISH

Mazkur maqola akademik litseylar Davlat ta’lim standartida belgilangan hamda matematik ta’lim mazmuniga kiritilgan algebraik shakldagi kompleks sonlar va ular ustida amallar,trigonometrik shakldagi kompleks sonlar va ular ustida amallar, kompleks sondan ildiz chiqarish mavzularini o’zlashtirishda o’quvchilarga va o’qituvchilarga metodik yordam sifatida tayyorlangan.

Ushbu maqolada yuqorida ko’rsatilgan mavzularga oid ko’pgina misollar yechimlari bilan berilgan.

Kompleks sonlar, ular ustida amallar, kompleks sonning moduli va argumenti, teng kompleks sonlar, o’zaro qo’shma kompleks sonlar, kompleks sonning trigonometrik shakli, trigonometrik shaklda berilgan kompleks sonlarni ko’paytirish, bo’lish, darajaga ko’tarish, kompleks sondan ildiz chiqarishga oid tushuncha va ma’lumotlar misollar yordamida yoritilgan.

Maqolaning har bir bobi bir nechta paragraflardan tashkil topgan bo’lib, har bir paragrafda mustaqil ishlash uchun misollar keltirilgan.

Maqolani yozishda mavjud adabiyotlar va darsliklardan foydalanildi.

I BOB.Algebraik shakldagi kompleks sonlar va ular ustida amallar.

1-§. $a + bi$ ko’rinishidagi ifoda.

Kompleks sonlar matematikada alohida o’rin tutadi. Tez rivojlanayotgan bu soha texnikada, shuningdek ishlab chiqarishning ko’plab sohalarida g’oyat keng qo’llanishga ega.

Xususiy bir misoldan boshlaylik.

$x^2 + 4 = 0$ tenglamani yechish jarayonida $x_1 = -2\sqrt{-1}$ va $x_2 = 2\sqrt{-1}$ “sonlar” hosil bo’ladi. Haqiqiy sonlar orasida esa bunday “sonlar” mavjud emas. Bunday holatdan qutulish uchun $\sqrt{-1}$ ga son deb qarash zarurati paydo bo’ladi.

Bu yangi son hech qanday real kattalikning o’lchamini yoki uning o’zgarishini ifodalamaydi. Shu sababli uni mavhum(xayoliy,haqiqatda mavjud bo’lmagan) birlik deb atash va maxsus belgilash qabul qilingan: $\sqrt{-1 = i}$. Mavhum birlik uchun $i^2 = -1$ tenglik o’rinlidir.

$a + bi$ ko’rinishidagi ifodani qaraymiz. Bu yerda a va b lar istalgan haqiqiy sonlar, i esa mavhum birlik. $a + bi$ ifoda haqiqiy son a va mavhum son bi lar “kompleksi” dan iborat bo’lgani uchun uni kompleks son deb atash qabul qilingan.

a+bi ifoda algebraik shakldagi kompleks son deb ataladi, bu yerda $a \in R$, $b \in R$, $i^2 = -1$. Bu paragrafda a+bi ni qisqalik uchun “algebraik shakldagi kompleks son” deyish o’rniga “kompleks son” deb ishlataveramiz.

Kompleks sonlarni bitta harf bilan belgilash qulay. Masalan, a+bi ni z=a+bi ko'rinishda belgilash mumkin. z=a+bi kompleks sonning haqiqiy qismi a ni Re(z) (fransuzcha reele-haqiqiy) bilan, mavhum qismi b ni esa Im(z) (fransuzcha imaginaire-mavhum) bilan belgilash qabul qilingan: a=Re(z) b=Im(z).

Agar z=a+bi kompleks son uchun b=0 bo'lsa, haqiqiy son z=a hosil bo'ladi.Demak, haqiqiy sonlar to'plami R barcha kompleks sonlar to'plami C ning qism to'plami bo'ladi: R⊂ C.

1-misol. $z_1 = -5 + 8i$, $z_2 = 1 - 3i$, $z_3 = 6$, $z_4 = 0$, $z_5 = 4i$ kompleks sonlarning haqiqiy va mavhum qismlarini toping.

Yechish: Kompleks son haqiqiy va mavhum qismlarining aniqlanishiga ko'ra,quyidagilarga egamiz:

$$\mathrm{Re}(z_1) = -5; \quad \mathrm{Re}(z_2) = 1; \quad \mathrm{Re}(z_3) = 6; \quad \mathrm{Re}(z_4) = 0; \quad \mathrm{Re}(z_5) = 0$$

$$\mathrm{Im}(z_1) = 8; \quad \mathrm{Im}(z_2) = -3; \quad \mathrm{Im}(z_3) = 0; \quad \mathrm{Im}(z_4) = 0; \quad \mathrm{Im}(z_5) = 4$$

Haqiqiy va mavhum qismlari mos ravishda teng bo'lgan kompleks sonlar teng kompleks sonlar deb ataladi.

Masalan, $z_1 = 1{,}5 + \frac{4}{5}i$ va $z_2 = \frac{3}{2} + 0{,}8i$ sonlari uchun $\mathrm{Re}(z_1) = \mathrm{Re}(z_2) = 1{,}5$, $\mathrm{Im}(z_1) = \mathrm{Im}(z_2) = 0{,}8$. Demak, $z_1 = z_2$.

Bir-biridan faqat mavhum qismlarining ishorasi bilan farq qiladigan ikki kompleks son o'zaro qo'shma kompleks sonlar deyiladi. z=a+bi kompleks songa qo'shma kompleks son $\bar{z} = a - bi$ ko'rinishida yoziladi. Masalan,6+7i va 6-7i lar qo'shma kompleks sonlardir: $\overline{6 + 7i} = 6 - 7i$.

3.1. Kompleks son z ning haqiqiy qismi Re(z) ni va mavhum qismi Im(z) ni toping:

a) z=−5+8i; f) z=0,5+3i; j) z=8i;

b) z=6+$\frac{1}{2}$i; g) z=2+0,3i; k) z=4;

d) z=−15+2i; h) z=−4,1+2i; l) z=0;

e) z=$\frac{1}{2} + \frac{3}{2}$i; i) z=−3−4i; m) z=−3i.

Yechish: Kompleks son haqiqiy va mavhum qismlarining aniqlanishiga ko'ra, quyidagilarga ega bo'lamiz:

a) Re(z)=−5; Im(z)=8

b) Re(z)=6; Im(z)=$\frac{1}{2}$.

d) Re(z)=−15; Im(z)=2.

e) Re(z)= $\frac{1}{2}$; Im(z) $= \frac{3}{2}$.

f) Re(z)=0,5; Im(z)=3.

g) Re(z)=2; Im(z)=0,3.

h) Re(z)=−4,1; Im(z)=2.

i) Re(z)=−3; Im(z)=−4.

j) Re(z)=0; Im(z)=8.

k) Re(z)=4; Im(z)=0.

l) Re(z)=0; Im(z)=0.

m) Re(z)=0; Im(z)=−3.

3.2. Agar:

a) Re(z)=−4, Im(z)=8;

b) Re(z)=0, Im(z)=1,2;

d) Re(z)=1,2, Im(z)=0;

e) Re(z)=0, Im(z)=0.

bo'lsa, z kompleks sonini algebraik shaklda yozing.

Yechish: z=a+bi ifodada a=Re(z) va b=Im(z) ga teng. Bunga ko'ra

a) z=−4+8i

b) z=1,2i;

d) z=1,2;

e) z=0.

3.3. Teng kompleks sonlarni toping:

$$\frac{1}{2}+\frac{1}{3}i;\quad 0{,}5+3i;\quad \frac{1}{4}+\frac{2}{6}i;\quad \sqrt{9}-4i;\quad \sqrt{9}-\sqrt{81}i;\quad 3-4i.$$

Yechish: Teng kompleks sonlar ta'rifiga ko'ra, ularning haqiqiy va mavhum qismlari o'zaro teng bo'lishi kerak. Yuqoridagi misolda

$\sqrt{9}-4i$ va $3-4i$ kompleks sonlari o'zaro teng.

3.4. Kompleks sonlardan qaysilari o'zaro teng:

$$3i;\ -4+5i;\ \frac{1}{3}+i;\ -\frac{1}{4}-8i;\ 0,(3)+i;\ -\frac{2}{8}-\sqrt{64}i;\ \sqrt[4]{81}i\ ?$$

Yechish: $3i=\sqrt[4]{81}i$; $\frac{1}{3}+i=0,(3)+i$; $-\frac{1}{4}-8i=-\frac{2}{8}-\sqrt{64}i.$

3.5. Agar: a) z=-3+5i; f) z=-3i; j) z=$\frac{1}{3}+3{,}4i$;

b) z=3-5i; g) z=4,2; k) z=0;

d) z=-3-4i; h) z=4i; l) z=$\sqrt{81}+4i$;

e) z=3+5i; i) z=4,(3)i; m) z=$-0,(3)-2,(3)i$.

bo'lsa, $\bar{z}$ ni toping.

Yechish:

a) $\bar{z}=-3-5i$; b) $\bar{z}=3+5i$; d) $\bar{z}=-3+4i$; e) $\bar{z}=3-5i$

f) $\bar{z}=3i$; g) $\bar{z}=4{,}2$; h) $\bar{z}=-4i$; i) $\bar{z}=-4,(3)i$

j) $\bar{z}=\frac{1}{3}-3{,}4i$; k) $\bar{z}=0$; l) $\bar{z}=\sqrt{81}-4i$; m) $\bar{z}=-0,(3)+2,(3)i.$

2-§. Kompleks sonlar ustida amallar.

Kompleks sonlar ustida amallar quyidagicha aniqlanadi:

(a+bi)+(c+di)=(a+c)+(b+d)i; (1)

(a+bi)-(c+di)=(a-c)+(b-d)i; (2)

(a+bi)·(c+di)=(ac-bd)+(ad+bc)i; (3)

$$\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i \qquad (4)$$

(1) va (2) tengliklarni bevosita qo'llash qiyin emas. Kompleks sonlarni ko'paytirish amalini $i^2 = -1$ ekanligini e'tiborga olib, ko'phadlarni ko'paytirish kabi bajarish mumkin.

1-misol. Ko'paytmani toping:

Qavslarni ochamiz va $i^2 = -1$ ekanidan foydalanamiz:

$$(2 - i) \cdot \left(\frac{3}{4} + 2i\right) = 2 \cdot \frac{3}{4} + 2 \cdot 2i - i \cdot \frac{3}{4} - 2i^2 =$$

$$= \frac{3}{2} + 4i - \frac{3}{4}i + 2 = \frac{7}{4} + \frac{13}{4}i\,.$$

(4) formulani eslab qolish va amaliyotda bevosita qo'llash ancha qiyin. Shu sababli $\frac{a+bi}{c+di}$ ni hisoblash uchun, uning surati va maxrajini $c - di$ ga ko'paytirib, tegishli amallarni bajarish qulaydir.

2-misol. Bo'lish amalini bajaring: $\frac{2-i}{-3+2i} = \frac{(2-i)(-3-2i)}{(-3+2i)(-3-2i)} = \frac{-6-4i+3i-2}{9+6i-6i+4} = \frac{-8-i}{13} = \frac{-8}{13} - \frac{1}{13}i$

3-misol. Yig'indini hisoblang: $(3 + 7i) + (-5 + 4i)$.

Yig'indini topish uchun (1) formuladan foydalanamiz:

$$(3 + 7i) + (-5 + 4i) = \big(3 + (-5)\big) + (7 + 4)i = -2 + 11i.$$

4-misol. Ayirmani toping: $(13 - 7i) - (-5 + 4i)$.

Ayirmani topish uchun (2) formuladan foydalanamiz:

$$(13 - 7i) - (-5 + 4i) = \big(13 - (-5)\big) + (-7 - 4)i = 8 - 11i.$$

Kompleks sonlarni qo'shish va ko'paytirish amallari xossalari haqiqiy sonlarnikiga o'xshash:

1) $z+w=w+z$;

2) $(z+w)+t=z+(w+t)$;

3) $z+0=z$;

4) $z(w+t)=zw+zt$.

3.6. Yig'indini hisoblang:

a) $(-3 + 2i) + (4 - i)$;

b) $(4 + 5i) + (4 - 5i)$;

d) $(5 + 2i) + (-5 - 2i)$;

e) $4 + (-3 + i)$;

f) $(1{,}4 - 3i) + (2{,}6 - 4i)$;

g) $(3 + 8i) + (3 - 8i)$;

h) $(-7 + 3i) + (7 - 3i)$;

i) $4{,}3 + (1{,}7 - 9i)$;

j) $8i + (4 - 6i)$;

k) $-15i + (-4 + 5i)$;

l) $(14+2i)+8i$;

m) $81 + (43 - 17i)$.

Yechish: Yig'indini topish uchun (1) formuladan foydalanamiz:

a) $(-3 + 2i) + (4 - i) = (-3 + 4) + (2 - 1)i = 1 + i$;

b) $(4 + 5i) + (4 - 5i) = (4 + 4) + (5 + (-5))i = 8$;

d) $(5 + 2i) + (-5 - 2i) = (5 + (-5)) + (2 + (-2))i = 0$;

e) $4 + (-3 + i) = (4 + (-3)) + i = 1 + i$;

f) $(1{,}4 - 3i) + (2{,}6 - 4i) = (1{,}4 + 2{,}6) + \big(-3 + (-4)\big)i = 4 - 7i$;

g) $(3 + 8i) + (3 - 8i) = (3 + 3) + \big(8 + (-8)\big)i = 6$.

h) $(-7+3i)+(7-3i)=(-7+7)+\big(3+(-3)\big)i=0$;

i) $4{,}3+(1{,}7-9i)=(4{,}3+1{,}7)+\big(0+(-9)\big)i=6+(-9)i=6-9i$;

j) $8i+(4-6i)=(0+4)+\big(8+(-6)\big)i=4+2i$;

k) $-15i+(-4+5i)=\big(0+(-4)\big)+(-15+5)i=-4-10i$;

l) $(14+2i)+8i=(14+0)+(2+8)i=14+10i$;

m) $81+(43-17i)=(81+43)+\big(0+(-17)\big)i=124-17i$.

3.7. Yig'indini toping:

a) $\left(\frac{1-\sqrt{2}}{2}+\frac{1+\sqrt{2}}{3}i\right)+\left(\frac{1+\sqrt{2}}{2}+\frac{1-\sqrt{2}}{3}i\right)$

b) $(\cos^2\alpha+i\sin^2\alpha)+(\sin^2\alpha+i\cos^2\alpha), \alpha\in R$;

d) $\big(0{,}3+i\cdot 1,(5)\big)+\big(0,(6)+i\cdot 1,(55)\big)$;

e) $(\mathrm{Re}(1+2i)+15i)+\big(3-i\cdot \mathrm{Im}(1+2i)\big)$.

Yechish: Yig'indini topish uchun (1) formuladan foydalanamiz:

a) $\left(\frac{1-\sqrt{2}}{2}+\frac{1+\sqrt{2}}{3}i\right)+\left(\frac{1+\sqrt{2}}{2}+\frac{1-\sqrt{2}}{3}i\right)=\left(\frac{1-\sqrt{2}}{2}+\frac{1+\sqrt{2}}{2}\right)+\left(\frac{1+\sqrt{2}}{3}++\frac{1-\sqrt{2}}{3}\right)i=$ $\left(\frac{1-\sqrt{2}+1+\sqrt{2}}{2}\right)+\left(\frac{1+\sqrt{2}+1-\sqrt{2}}{3}\right)i=\frac{2}{2}+\frac{2}{3}i=1+\frac{2}{3}i$;

b) $(\cos^2\alpha+i\sin^2\alpha)+(\sin^2\alpha+i\cos^2\alpha)=(\cos^2\alpha+\sin^2\alpha)+(\sin^2\alpha+$ $+\cos^2\alpha)i=1+i$.

d) $\big(0{,}3+i\cdot 1,(5)\big)+\big(0,(6)+i\cdot 1,(55)\big)=\big(0{,}3+0,(6)\big)++\big(1,(5)+$ $+1,(55)\big)i=\left(\frac{3}{10}+\frac{6}{9}\right)+\left(1\frac{5}{9}+1\frac{55}{99}\right)i=\frac{29}{30}+\frac{308}{99}i$;

e) Bu misolni ishlashda kompleks sonning haqiqiy va mavhum qismini topamiz, ya'ni $\mathrm{Re}(1+2i)=1$; $\mathrm{Im}(1+2i)=2$.

$$(\mathrm{Re}(1+2i)+15i)+\big(3-i\cdot\mathrm{Im}(1+2i)\big)=(1+15i)+(3-2i)$$
$$=(1++3)+\big(15+(-2)\big)i=4+13i.$$

3.8. Ayirmani toping:

a) $(-5+2i)-(8-9i)$;

b) $(5+21i)-(9i+8)$;

d) $\big(4-(42-3i)\big)$;

e) $(14+3i)-(21+3i)$;

f) $(32+4,(5)i)-(32+i)$;

g) $\left(\frac{1-\sqrt{2}}{2}+\frac{1-\sqrt{2}}{2}i\right)-(1+i)$;

h) $4{,}8-\left(\frac{1-\sqrt{2}}{3}-i\right)$;

i) $i-(3i+8)$.

Yechish: Bu misollarni ishlash uchun (2) formuladan foydalanamiz:

a) $(-5+2i)-(8-9i)=((-5-8)+\big(2-(-9)\big)i=-13+11i$;

b) $(5+21i)-(9i+8)=(5-8)+(21-9)i=-3+12i$;

d) $\big(4-(42-3i)\big)=4-42+3i=-38+3i$;

e) $(14+3i)-(21+3i)=(14-21)+(3-3)i=-7$.

f) $(32 + 4,(5)i) - (32 + i) = (32 - 32) + (4,(5) - 1)i = 0 + \left(4\frac{5}{9} - 1\right)i == \frac{32}{9}i$;

3.9. Ko'paytmani hisoblang:

a) $(\mathbf{3 + 5i})(\mathbf{2 + 3i})$;
b) $(\mathbf{4 + 7i})(\mathbf{2 - i})$;
d) $(\mathbf{5 - 3i})(\mathbf{2 - 5i})$;
e) $(\mathbf{-2 + i})(\mathbf{7 - 3i})$;
f) $\left(\frac{1}{2} + \mathbf{i}\right)\left(\frac{1}{4} - \mathbf{i}\right)$;
g) $\left(\frac{4}{7} + \mathbf{3i}\right)\left(\frac{7}{4} + \mathbf{4,7i}\right)$;
h) $(\mathbf{2 + 3i})(\mathbf{2 - 3i})$;
i) $\mathbf{4 \cdot (8,3 - i)}$;
j) $(\mathbf{5 - 2i})(\mathbf{2i + 5})$;
k) $(\mathbf{-3 + i})(\mathbf{3 - i})$;
l) $\mathbf{0 \cdot (4,5 - i)}$;
m) $\left(\frac{1}{3} - \mathbf{0,3}\right) \cdot \mathbf{i}$.

Yechish: Ikki kompleks sonning ko'paytmasi formulasidan foydalanamiz:

a) $(3 + 5i)(2 + 3i) = (3 \cdot 2 - 5 \cdot 3) + (3 \cdot 3 + 5 \cdot 2)i = -9 + 19i$;

b) $(4 + 7i)(2 - i) = \big(4 \cdot 2 - 7 \cdot (-1)\big) + (4 \cdot (-1) + 7 \cdot 2)i = 15 + 10i$;

d) $(5 - 3i)(2 - 5i) = 5 \cdot 2 - 5 \cdot 5i - 3i \cdot 2 + 3i \cdot 5i = 10 - 25i - 6i + 15i^2 =$
$= 10 - 25i - 6i - 15 = -5 - 31i$;

e) $(-2 + i)(7 - 3i) = -2 \cdot 7 + 2 \cdot 3i + 7i - i \cdot 3i = -14 + 6i + 7i - 3i^2 == -14 + 6i + 7i + 3 \cdot 1 = -11 + 13i$;

f) $\left(\frac{1}{2} + i\right)\left(\frac{1}{4} - i\right) = \frac{1}{2} \cdot \frac{1}{4} - \frac{1}{2} \cdot i + i \cdot \frac{1}{4} - i \cdot i = \frac{1}{8} - \frac{1}{2}i + \frac{1}{4}i + 1 === 1\frac{1}{8} - \frac{1}{4}i$;

g) $\left(\frac{4}{7} + 3i\right)\left(\frac{7}{4} + 4,7i\right) = \frac{4}{7} \cdot \frac{7}{4} + \frac{4}{7} \cdot \frac{47}{10}i + 3i \cdot \frac{7}{4} + 3i \cdot \frac{47}{10}i = 1 + \frac{94}{35}i + + \frac{21}{4}i + \frac{141}{10} \cdot i^2 = 1 + \frac{1111}{140}i - \frac{141}{10} = -\frac{131}{10} + \frac{1111}{140}i$;

h) $(2 + 3i)(2 - 3i) = 2 \cdot 2 - 2 \cdot 3i + 3i \cdot 2 - 3i \cdot 3i = 4 - 6i + 6i + 9 = 13$;

i) $4 \cdot (8,3 - i) = 4 \cdot 8,3 - 4 \cdot i = 33,2 - 4i$;

j) $(5 - 2i)(2i + 5) = 5 \cdot 2i + 5 \cdot 5 - 2i \cdot 2i - 2i \cdot 5 = 10i + 25 - 4i^2 - 10i =$
$= 10i + 25 + 4 - 10i = 29$;

k) $(-3 + i)(3 - i) = -3 \cdot 3 - 3 \cdot (-i) + i \cdot 3 - i \cdot i = -9 + 3i + 3i - i^2 === -9 + 3i + 3i + 1 = -8 + 6i$;

l) $0 \cdot (4,5 - i) = 0 \cdot 4,5 - 0 \cdot i = 0$;

m) $\left(\frac{1}{3} - 0,3\right) \cdot i = \frac{1}{3} \cdot i - 0,3 \cdot i = \frac{1}{3}i - \frac{3}{10}i = \frac{1}{30}i$.

3.10. Bo'linmani hisoblang:

a) $\frac{1+i}{1-i}$; **b**) $\frac{3-4i}{2+i}$; **d**) $\frac{2+3i}{2-3i}$;
e) $\frac{1 + 2i}{3 - 2i}$; **f**) $\frac{5 - 4i}{-3 + 2i}$; **g**) $\frac{-7 + 2i}{5 - 4i}$;
h) $\frac{3 - 4i}{-3 + 2i}$; **i**) $\frac{14 - 3i}{3i + 2}$; **j**) $\frac{51}{4 - i}$;

Yechish: Ikki kompleks sonning bo'linmasini topish formulasidan foydalanib ishlab chiqamiz, ya'ni ikki kompleks sonning bo'linmasini topish uchun berilgan kompleks sonning maxrajiga qo'shma kompleks sonni uning surati va maxrajiga ko'paytirib chiqamiz:

a) $\frac{1+i}{1-i}=\frac{(1+i)(1+i)}{(1-i)(1+i)}=\frac{1+i+i+i^2}{1+i-i-i^2}=\frac{1+i+i-1}{1+i-i+1}=\frac{2i}{2}=\mathbf{i}$;

b) $\frac{3-4i}{2+i}=\frac{(3-4i)(2-i)}{(2+i)(2-i)}=\frac{6-3i-8i+4i^2}{4-2i+2i-i^2}=\frac{6-11i-4}{4+1}=\frac{2-11i}{5}==\frac{2}{5}-\frac{11}{5}\mathbf{i}$;

d) $\frac{2+3i}{2-3i}=\frac{(2+3i)(2+3i)}{(2-3i)(2+3i)}=\frac{4+6i+6i+9i^2}{4+6i-6i-9i^2}=\frac{4+12i-9}{4+9}=\frac{-5+12i}{13}==\frac{-5}{13}+\frac{12}{13}\mathbf{i}$;

e) $\frac{1+2i}{3-2i}=\frac{(1+2i)(3+2i)}{(3-2i)(3+2i)}=\frac{3+2i+6i+4i^2}{9+6i-6i-4i^2}=\frac{3+8i-4}{9+4}=\frac{-1+8i}{13}==\frac{-1}{13}+\frac{8}{13}\mathbf{i}$;

f) $\frac{5-4i}{-3+2i}=\frac{(5-4i)(-3-2i)}{(-3+2i)(-3-2i)}=\frac{-15-10i+12i+8i^2}{9+6i-6i-4i^2}=\frac{-15+2i-8}{9+4}==\frac{-23+2i}{13}=\frac{-23}{13}+\frac{2i}{13}$;

g) $\frac{-7+2i}{5-4i}=\frac{(-7+2i)(5+4i)}{(5-4i)(5+4i)}=\frac{-35-28i+10i+8i^2}{25+20i-20i-16i^2}=\frac{-35-18i-8}{25+16}==\frac{-43-18i}{41}=\frac{-43}{41}-\frac{18i}{41}$;

h) $\frac{3-4i}{-3+2i}=\frac{(3-4i)(-3-2i)}{(-3+2i)(-3-2i)}=\frac{-9-6i+12i+8i^2}{9+6i-6i-4i^2}=\frac{-9+6i-8}{9+4}==\frac{-17+6i}{13}=\frac{-17}{13}+\frac{6i}{13}$;

i) $\frac{14-3i}{3i+2}=\frac{(14-3i)(2-3i)}{(2+3i)(2-3i)}=\frac{28-42i-6i+9i^2}{4-6i+6i-9i^2}=\frac{28-48i-9}{4+9}===\frac{19-48i}{13}=\frac{19}{13}-\frac{48}{13}\mathbf{i}$;

j) $\frac{51}{4-i}=\frac{51(4+i)}{(4-i)(4+i)}=\frac{204+51i}{16+4i-4i-i^2}=\frac{204+51i}{16+1}=\frac{204+51i}{17}==\frac{204}{17}+\frac{51}{17}\mathbf{i}$.

3.13. Amallarni bajaring:

a) $\mathbf{-3+5+8i(3-i)=-3+5+24i-8i^2=2+24i+8=10+24i}$;

b) $\mathbf{(4+2i)(-1-3i)+5-8i=-4-12i-2i-6i^2+5-8i=-4-14i+}$
$\mathbf{+6+5-8i=7--22i}$;

d) $\mathbf{3i(1+i)+3i(3-i)=3i+3i^2+9i-3i^2=3i-3+9i+3=12i}$;

e) $\frac{1}{2}\mathbf{i(5-2i)}+\frac{1}{3}\mathbf{i(9-8i)}=\frac{5}{2}\mathbf{i-i^2+3i}-\frac{8}{3}\mathbf{i^2}=\frac{11}{3}+\frac{11}{2}\mathbf{i}$;

f) $\mathbf{(5-3i)(4+i)+15i=20+5i-12i-3i^2+15i=23+8i}$;

g) $\mathbf{16-(15-i)(1+i)=16-15-15i+i+i^2=-14i}$;

3.14. Hisoblang:

a) $\frac{(2-3i)(3-2i)}{1+i}=\frac{6-4i-9i+6i^2}{1+i}=\frac{-13i}{1+i}=\frac{-13i(1-i)}{(1+i)(1-i)}=\frac{-13i+13i^2}{1-i+i-i^2}==\frac{-13-13i}{2}$;

b) $\frac{(3-i)(1+3i)}{2-i}=\frac{3+9i-i-3i^2}{2-i}=\frac{3+8i+3}{2-i}=\frac{(6+8i)(2+i)}{(2-i)(2+i)}=\frac{12+6i+16i+8i^2}{4+2i-2i-i^2}=\frac{12+22i-8}{4+1}=$
$\frac{4+22i}{5}=\frac{4}{5}+\frac{22i}{5}$;

d) $\frac{3-4i}{(1+i)(2-i)}=\frac{3-4i}{2-i+2i-i^2}=\frac{3-4i}{3+i}=\frac{(3-4i)(3-i)}{(3+i)(3-i)}=\frac{9-3i-12i+4i^2}{9-3i+3i-i^2}==\frac{5-15i}{10}=\frac{1}{2}-\frac{3i}{2}$;

e) $\frac{2-3i}{(1-i)(3+i)}=\frac{2-3i}{3+i-3i-i^2}=\frac{2-3i}{4-2i}=\frac{(2-3i)(4+2i)}{(4-2i)(4+2i)}=\frac{8+4i-12i-6i^2}{16+8i-8i-4i^2}=\frac{14-8i}{20}=\frac{7}{10}-\frac{2i}{5}$;

f) $\frac{11}{1-2i}-\frac{13}{2-i}=\frac{11(1+2i)}{(1-2i)(1+2i)}-\frac{13(2+i)}{(2-i)(2+i)}=\frac{11(1+2i)}{1+2i-2i-4i^2}--\frac{13(2+i)}{4+2i-2i-i^2}=\frac{11+22i}{5}-$
$-\frac{26+13i}{5}=\frac{11+22i-26-13i}{5}=\frac{-15+9i}{5}=\mathbf{-3+1,8i}$;

g) $\frac{13}{1-4i}+\frac{11}{1+4i}=\frac{13(1+4i)}{(1-4i)(1+4i)}+\frac{11(1-4i)}{(1+4i)(1-4i)}=\frac{13+52i}{1+4i-4i-16i^2}++\frac{11-44i}{1-4i+4i-16i^2}=\frac{13+52i}{17}+$
$\frac{11-44i}{17}=\frac{13+52i+11-44i}{17}=\frac{24+8i}{17}=\frac{24}{17}+\frac{8i}{17}$.

3.15. Amallarni bajaring:

a) $\mathbf{(3-2i)^2=9-12i+4i^2=9-12i-4=5-12i}$;

b) $\mathbf{(4+3i)^2=16+24i+9i^2=16+24i-9=7+24i}$;

d) $\left(\frac{\mathbf{1-2i}}{\mathbf{1+i}}\right)^{\mathbf{2}} = \frac{(\mathbf{1-2i})^{\mathbf{2}}}{(\mathbf{1+i})^{\mathbf{2}}} = \frac{1-4i+4i^2}{1+2i+i^2} = \frac{-3-4i}{2i} = \frac{(-3-4i)(-2i)}{2i(-2i)} = \quad = \frac{6i+8i^2}{-4i^2} = \frac{6i-8}{4} = -2 + \frac{3i}{2}$;

e) $\left(\frac{1+i}{1-i}\right)^2 = \frac{(1+i)^2}{(1-i)^2} = \frac{1+2i+i^2}{1-2i+i^2} = \frac{2i}{-2i} = -1$;

f) $(3+2i)^2 - (3-2i)^2 = 9 + 12i + 4i^2 - 9 + 12i - 4i^2 = 24i$;

g) $(-3+5i) + (-3-5i) = -3 + 5i - 3 - 5i = -6$;

h) $(\frac{i+1}{i-1})^2 = \frac{(i+1)^2}{(i-1)^2} = \frac{i^2+2i+1}{i^2-2i+1} = \frac{2i}{-2i} = -1$;

i) $(\frac{4+i}{3-i})^2 = \frac{(4+i)^2}{(3-i)^2} = \frac{16+8i+i^2}{9-6i+i^2} = \frac{15+8i}{8-6i} = \frac{(15+8i)(8+6i)}{(8-6i)(8+6i)} = = \frac{120+90i+64i+48i^2}{64+48i-48i-36i^2} = \frac{72+154i}{100} = \frac{18}{25} + \frac{77i}{50}$.

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